**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Ans… time to service a car is 50 min,**

**P(X>50) = 1-P(X≤50)**

**Z=(X-45)/8**

**Therefore,**

**P(X≤50)=P(Z≤(50-45)/8)**

**=P(Z≤0.625)**

**=73.24%**

**Therefore, the service manager cannot meet his commitment**

**=100-73.24**

**=26.76% = 0.2676**

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

**Ans… False, if more employees are older than 44 then mean will shift towards 44 with**

**considering sd, which is not possible as it is given with mean 38 and sd 6.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans… True**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans… As we know that if X ∼ N(µ1, σ1^2), and Y ∼ N(µ2, σ2^2) are two independent random**

**variables ,**

**then, X+Y ∼ N(µ1+µ2, σ1^2+σ2^2), and X−Y ∼ N(µ1−µ2, σ1^2+σ2^2).**

**Similarly, if Z =aX+bY, where X and Y are as defined above,**

**i.e, Z is linear combination of X and Y,**

**then, Z ∼ N(aµ1+bµ2, a^2σ1^2+b^2σ2^2).**

**Therefore in the question,**

**2X1~ N(2µ, 4σ^2) and**

**X1+X2 ~ N(µ+µ, σ^2+σ^2) ~ N(2µ, 2σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

**Ans… qnorm(0.995,100,20)**

**=151.5166**

**qnorm(0.005,100,20)**

**=48.48341**

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. **48.5, 151.5**
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans… qnorm(0.025,45\*5,3)**

**=219.1201**

**qnorm(0.975,45\*5,3)**

**=230.8799**

**qnorm(0.025,45\*7,4)**

**=307.1601**

**qnorm(0.975,45\*7,4)**

**=322.8399**

**Rupee range such that it contains 95% probability for the annual profit of the**

**company**

**=P1+P2**

**=[219.12,230.88] + [307.16,322.84]**

**=[526.28,553.72]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans… qnorm(0.05,45\*7,4)**

**=308.4206**

**qnorm(0.05,45\*5,3)**

**=220.0654**

**5th percentile of profit for the company**

**=308.42 + 220.06**

**=528.48**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans… division 2 with distribution N(7, 42)** **has a larger probability of making a loss in a**

**given year.**